

If the farmer can plant the trees in a square grid, he has a number of trees that is a perfect square.

Let the number of trees the farmer bought be x .

$x+20$ is a perfect square since if he had 20 more trees, he could plant them in a square grid. Let it be n^2 . Similarly, $x-39$ is a perfect square.

Let it be m^2 .

Now you have two equations:

$$n^2 = x + 20$$

$$m^2 = x - 39$$

Subtracting the two equations, you get:

$$n^2 - m^2 = 59$$

$$(n+m)(n-m) = 59$$

$n+m$ and $n-m$ must be integers, so:

$$\begin{aligned} n+m &= 59 \\ n-m &= 1 \end{aligned}$$

$$\text{or} \quad \begin{aligned} n+m &= 1 \\ n-m &= 59 \end{aligned}$$

The second system of equations yields a negative answer, so you must use the first one.

$$\text{Solving,} \quad \begin{aligned} n &= 30 \\ m &= 29 \end{aligned}$$

You can now use either n or m to find x .

Since $n^2 = x + 20$, you get that x must be $\boxed{880}$.